Published by Institute of Physics Publishing for SISSA
Received: *December 5, 2006*
Revised: *February 19,2007*
Accepted: *March 7, 2007*
Published: *March 15, 2007*

Massive IIA flux compactifications and U-dualities

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Abstract: We attempt to find a rigorous formulation for the massive type IIA orientifold compactifications of string theory introduced in [1]. An approximate double T-duality converts this background into IIA string theory on a twisted torus, but various arguments indicate that the back reaction of the orientifold on this geometry is large. In particular, an AdS calculation of the entropy suggests a scaling appropriate for N M2-branes, in a certain limit of the compactification, though not the one studied in [1]. The M-theory lift of this specific regime is not 4 dimensional. We suggest that the generic limit of the background corresponds to a situation analogous to F-theory, where the string coupling is small in some regions of a compact geometry, and large in others, so that neither a long wavelength 11D SUGRA expansion, nor a world sheet expansion exists for these compactifications. We end with a speculation on the nature of the generic compactification.

Keywords: String Duality, Superstring Vacua, Flux compactifications.

http://jhep.sissa.it/archive/papers/jhep032007068/jhep032007068.pdf

JHEP03(2007)068

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1. Introduction

Flux compactifications [2] provide the arena for most of the discussions of the String Landscape as well as modern approaches to string phenomenology. The discussion of these compactifications is generally carried out in low energy effective field theory [3, 4], despite the fact that they all contain orientifold singularities. Further, there is no perturbative world sheet treatment of these backgrounds. Recently, DeWolfe et al.[1] introduced a sequence of models characterized by an integer N. Earlier work on similar type IIA flux compactifications was done in [5]. The DeWolfe et al. compactifications are classical solutions of Type IIA SUGRA, with a singular orientifold source and a variety of Ramond-Ramond and Neveu Schwarz fluxes. The parameter N is related to the value of certain quantized fluxes, and may be taken arbitrarily large. This is in marked contrast to typical flux compactifications, where fluxes are bounded [6]. The authors of [1] argue that for large N the moduli can be stabilized at values where all radii are large compared to the string scale, and the string coupling is small. Furthermore, the four non-compact directions are an AdS space with a radius R_{AdS} whose ratio to the compactification scale grows with N. The latter property is in marked contrast to the sequences of models treated in the AdS/CFT correspondence.

Our aim in this paper is to investigate further the models of [1], and to determine whether they admit a systematic low energy field theory expansion (see also [7]) and/or a weakly coupled string expansion. The inevitable orientifold of flux compactifications is one potential barrier to an effective field theory treatment.[1] In addition, these models contain a ten form flux F_0, and correspond to solutions of the massive Type IIA SUGRA Lagrangian. It is well known that quantization of F_0 is a problem for effective field theory, and that the massive Type IIA string theory does not have a perturbative world sheet expansion (the D8-brane solution of this theory has a string coupling which grows at infinity). In addition, the effective field theory treatment has the usual problem of orientifold singularities. Thus despite apparently small parameters, it is far from clear that there is a systematic large N expansion of this system.

We approach this problem indirectly. Ignoring the back reaction of the orientifold, we perform a double T-duality on the DeWolfe et al. background.[2] The result is Type IIA string theory on a twisted torus, with flux only in the four AdS directions. Despite the fact that this configuration does not satisfy tadpole cancellation, the T-duality is a legitimate operation on the orbifold CFT. We then restore tadpole cancellation in the T-dual picture (the formal dual of the original orientifold).

Regime with one large 4-flux: We argue that the resulting model in this regime does not have a weakly coupled Type IIA world sheet expansion. In this limit, from the point of view of DeWolfe et al., the string coupling remains weak, the scales of both AdS_4 and

[1] G. Moore and S. Ramanujam have emphasized to us the problems with the back reaction of the orientifold, which they have analyzed extensively in the context of the original DeWolfe et al. solutions [8].

[2] We work in the orbifold limit. The authors of [1] took pains to show that the blow up moduli of the orbifold can be stabilized at large values of the radii of shrinking cycles. We address the analogous question in the T-dual picture.

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the compact manifold are large, and the Kaluza-Klein radius is parametrically smaller. However, some cycles on the compact manifold shrink to zero size, and this is not a limit in which DeWolfe et al. would claim to have a controlled expansion. In the limit where we only turn on one four form flux, the fixed temperature entropy computed from the AdS geometry scales like $N^{3/2}$ as one would expect from a large number of M2-branes. We show that this is explained in the T-dual IIA picture by a large number of D2-branes sitting at the orientifold locus, where the string coupling is large. The D2/M2 world volumes are in the AdS directions. In typical orientifold compactifications that have been studied in string theory, those with a known world sheet expansion, the effect of the orientifold is confined to a region of order string scale. Here we argue that this is not the case, since the parameter N, which apparently tunes the string coupling to be small, in fact counts a large number of branes near the orientifold singularity. We argue that in fact the strong coupling region completely dominates the geometry in the single flux limit. The resulting theory for large N is M-theory on $AdS_4 \times M_7$, where M_7 is a manifold of weak G_2 holonomy. The AdS and compact radii scale the same way with N.

Regime with all 4-flux large: For the generic regime of the background, we find that 11D SUGRA is not a valid approximation. This is a consequence of the small string coupling found by DeWolfe et al., combined with the observation that the AdS radius is much larger than that of the compact manifold computed using our naive T-duality rules. Thus, in this region where DeWolfe et al. claimed a systematic expansion, many features of their picture are valid. However, our picture also includes large numbers of D-branes sitting at the orientifold locus in the regime where all fluxes are large. We argue that the weak coupling approximation breaks down in a vicinity of the orientifold whose size scales like $N^{1/20} l_s$. This rules out a uniform weak coupling expansion in the large N limit. Furthermore, if we apply 11D SUGRA to the region around the orientifold, it suggests that this region actually blows up to a seven manifold whose radius of curvature is of order the AdS radius. In the conclusions, we also provide a heuristic explanation of the peculiar $N^{9/2}$ entropy scaling of the regime with all fluxes large. This argument also seems to require a compact manifold with volume much larger than that suggested by De Wolfe et al..

Our conclusion is that the generic DeWolfe et al. configuration probably exists as a valid model of quantum gravity in AdS_4. However, it is unclear to us whether is has a compactification radius parametrically smaller than the AdS radius. No existing approximation scheme computes its large N expansion. Different approximations, apparently valid in different regions of the compact manifold suggest different values for the ratio of scales. The problem of different approximation schemes for different regions is somewhat analogous to F-theory solutions for fluxless compactifications. However, the large supersymmetry algebra of F-theory compactifications provides reliable computational tools, which are absent for these models.

The paper is organized as follows. In section 2, we review the DeWolfe et al. background. In section 3, we transform the background by a double T-duality, using the approximations noted above. This allows us to eliminate the massive type IIA flux. We also comment on the approximate character of the transformation. Section 4 deals with the Bianchi condition for the dualized background. In section 5, we will argue that the DeWolfe et al. solution with one large flux should be considered in an M-theory setting. We will explicitly lift the dualized background to M-theory. Section 6 will detail some of the aspects of the obtained 11D SUGRA solution. We will discuss its interpretation as a stack of M2-branes. We conclude in section 7 where we speculate on the nature of the generic DeWolfe et al. compactification. Appendix A and B give some more details on the double T-duality transformation of the DeWolfe et al. background, while appendix C reviews the formulas to lift the background to M-theory.

2. The DeWolfe et al. background

2.1 The metric, fluxes and discrete symmetries of the solution

In [1], DeWolfe et al. describe an infinite set of $\mathcal{N} = 1$ solutions of massive type IIA SUGRA [9]. The compact manifold in their solution is $T^2 \times T^2 \times T^2$, modded out by three discrete symmetries:

- $\Omega_p(-1)^{F_L}\sigma$ with $\sigma : z_i \to -\bar{z}_i$
- $T : (z_1, z_2, z_3) \to (\alpha^2 z_1, \alpha^2 z_2, \alpha^2 z_3)$
- $Q : (z_1, z_2, z_3) \to (\alpha^2 z_1 + \frac{1+\alpha}{3}, \alpha^4 z_2 + \frac{1+\alpha}{3}, z_3 + \frac{1+\alpha}{3})$

with $\alpha = e^{2\pi i/6}$. The resulting space is orientifolded $T^6/\mathbb{Z}_3^2$. The combination of the imposed discrete symmetries and fluxes turned on leads to a background where all moduli are fixed. The metric and the fluxes of the background are given by,

$$\begin{aligned}
ds^2 &= \gamma_1(dx_1^2 + dx_2^2) + \gamma_2(dx_3^2 + dx_4^2) + \gamma_3(dx_5^2 + dx_6^2) + ds^2_{\mathrm{AdS}_4} && (2.1)\\
H_3 &= -4\pi^2\alpha' h_3\,\beta_0 && (2.2)\\
&= -4\pi^2\alpha' h_3 \sqrt[4]{3}\sqrt{2}\,(dx_1\wedge dx_3\wedge dx_5 - dx_1\wedge dx_4\wedge dx_6 \\
&\qquad - dx_2\wedge dx_3\wedge dx_6 - dx_2\wedge dx_4\wedge dx_5) && (2.3)\\
e^{\varphi} &= \frac{1}{4}|h_3|\sqrt[4]{\frac{3^3 5}{|f_0 f_4^1 f_4^2 f_4^3|}} && (2.4)\\
F_4 &= (2\pi\sqrt{\alpha'})^3\sqrt[3]{\kappa} f_4^i \tilde{w}^i && (2.5)\\
&= 4(2\pi\sqrt{\alpha'})^3\sqrt[3]{3}\Big(f_4^1\,dx_3\wedge dx_4\wedge dx_5\wedge dx_6 \\
&\qquad + f_4^2\,dx_5\wedge dx_6\wedge dx_1\wedge dx_2 && (2.6)\\
&\qquad + f_4^3\,dx_1\wedge dx_2\wedge dx_3\wedge dx_4\Big) \\
F_2 &\approx 0 && (2.7)\\
F_0 &= \frac{f_0}{2\pi\sqrt{\alpha'}}\,, && (2.8)
\end{aligned}$$

where $f_0, h_3, f_4^1, f_4^2, f_4^3 \in \mathbb{Z}$; $z_1 = x_1 + i x_2, \ldots$ and

$$\gamma_i = 4\pi^2\alpha' \frac{2}{\sqrt[3]{3}}\sqrt{\frac{5|f_4^1 f_4^2 f_4^3|}{|f_0|}\frac{1}{|f_4^i|}}\,. \qquad (2.9)$$

The tadpole cancellation condition for F_2 reduces to $f_0 h_3 = -2$. Notice that we defined the R-R fluxes following the conventions of [10] instead of those in [1] (see footnote 3 in [1] v3). The non-compact part of the metric, $ds^2_{\mathrm{AdS}_4}$, is a 4-dimensional AdS_4 space with radius (in string frame),

$$R^2_{\mathrm{AdS}} = 4\pi^2\alpha'\, 16\sqrt{\frac{5^3|f_4^1 f_4^2 f_4^3|}{3^3|f_0|^3|h_3|^4}}\,. \tag{2.10}$$

The volume of the compact manifold is computed to be

$$\mathrm{vol}_6 = \frac{1}{8\sqrt{3}}\gamma_1\gamma_2\gamma_3 \tag{2.11}$$

$$= (4\pi^2\alpha')^3\sqrt{\frac{5^3|f_4^1 f_4^2 f_4^3|}{3^3|f_0|^3}}\,, \tag{2.12}$$

where the factor

$$\frac{1}{8\sqrt{3}} = \left(\frac{1}{9^{1/6}}\right)^3\left(\frac{\sqrt{3}}{9^{1/6}2}\right)^3, \tag{2.13}$$

comes from the discrete identifications in the compact manifold. The four dimensional Planck length then becomes:

$$l_{P(4)} = \frac{1}{\sqrt{16\pi}}\left(\frac{\mathrm{vol}_6}{2\kappa_{10}^2 e^{2\varphi}}\right)^{-1/2} \tag{2.14}$$

$$= |h_3|\sqrt{\frac{3^3\alpha'}{2^7 5}\frac{|f_0|}{|f_4^1 f_4^2 f_4^3|}}\,, \tag{2.15}$$

where we used the convention $2\kappa_{10}^2 = (2\pi)^7\alpha'^4$.

2.2 The orientifold in the DeWolfe et al. background

The orientifold, constructed by modding out by $\Omega_p(-1)^{F_L}\sigma$, lies on

$$x_1 = x_3 = x_5 = 0\,. \tag{2.16}$$

Taking into account the identifications under $\mathbb{Z}_3\times\mathbb{Z}_3$, we get the three dimensional surface along which the orientifold is extended in the compact space. In figure 1, this surface is pictured in the fundamental region of one of the tori of $T^2\times T^2\times T^2$. The orientifold also fills the non-compact space.

The 3-cycle that is invariant under $\mathbb{Z}_2\times\mathbb{Z}_3\times\mathbb{Z}_3$, is the cycle on which the orientifold is wrapped. This cycle α_0, is determined by its Poincaré dual 3-form,

$$\mathrm{O6}\ :\ \alpha_0\text{−cycle} \tag{2.17}$$

$$:\ \sqrt[4]{3}\sqrt{2}\,(dx_1\wedge dx_3\wedge dx_5 - dx_2\wedge dx_4\wedge dx_5$$

$$- dx_1\wedge dx_4\wedge dx_6 - dx_2\wedge dx_3\wedge dx_6) \tag{2.18}$$

$$= \beta_0\,. \tag{2.19}$$

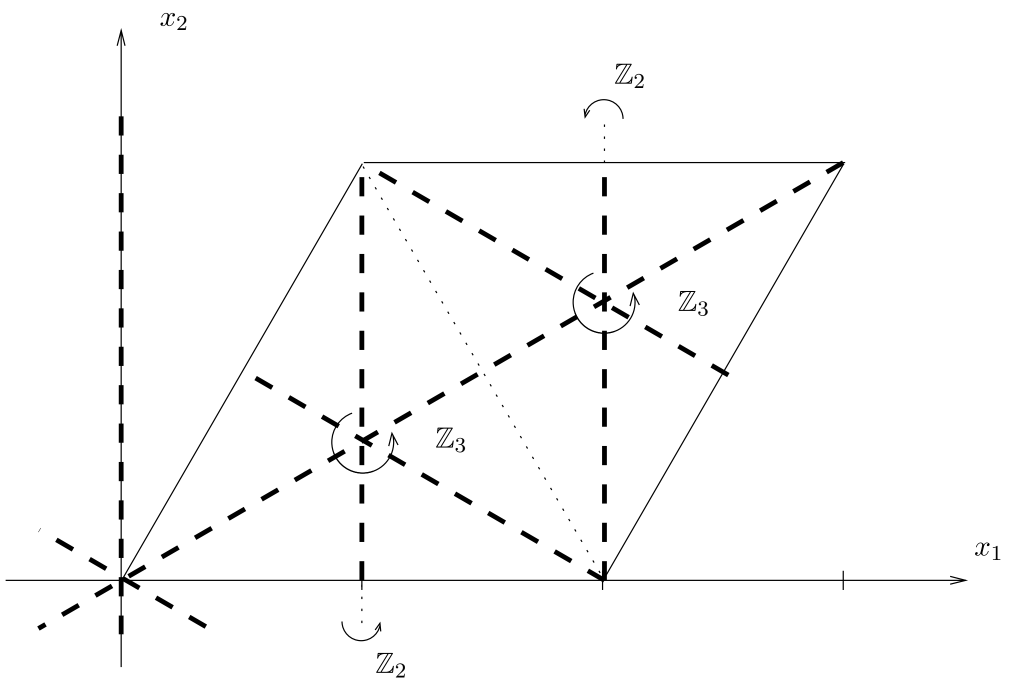

Figure 1: The z_1-plane of T^6 with the actions of the non-free $\mathbb{Z}_3$ and the orientifolding $\mathbb{Z}_2$ indicated. The O6-plane is pictured in thicker, dashed lines.

2.3 Remark on the F_2-flux

The Bianchi identity for the massive type IIA solution reads:

$$dF_2 = F_0 H_3 + 2\kappa_{10}^2 \mu_p \delta_{\rm O6} \neq 0\,, \tag{2.20}$$

where $\mu_p = -2\sqrt{\pi}\kappa_{10}^{-1}(4\pi^2\alpha')^{-3/2}$, is the charge of the orientifold. Equation (2.7),

$$F_2 = 0 \tag{2.21}$$

should thus be seen as an approximation to the exact solution.

2.4 Scaling behavior

The integer parameters f_4^1, f_4^2 and f_4^3 are not constrained by any tadpole condition, but we need to take each $f_4^i \neq 0$, to have a non-degenerate solution (see (2.9)).

We will be interested in the regime

$$f_4^1 = f_4^2 = f_4^3 = N\,, \tag{2.22}$$

where we take $N \to \infty$. The parameters characterizing the compactification scale as:

$$l_{P(4)} \sim N^{-\frac{3}{2}}\sqrt{\alpha'} \tag{2.23}$$

$$R_{\rm AdS} \sim N^{\frac{9}{4}} l_{P(4)} \tag{2.24}$$

$$R_{\rm KK} \sim N^{\frac{7}{4}} l_{P(4)} \tag{2.25}$$

$$g_s \sim N^{-\frac{3}{4}}\,, \tag{2.26}$$

where $R_{\rm KK} = \sqrt[6]{\rm vol_6}$ is a measure for the size of the compact manifold. We see that the string coupling g_s is small, while the radii characterizing the solution are large. In addition, we notice that the background remains effectively four dimensional since the AdS radius grows faster than the Kaluza Klein radius.

Let us now consider the regime where:

$$f_4^1 = N, \qquad f_4^2 = f_4^3 = O(1)\,, \tag{2.27}$$

which results in,

$$l_{P(4)} \sim N^{-\frac{1}{2}}\sqrt{\alpha'} \tag{2.28}$$

$$R_{\rm AdS} \sim N^{\frac{3}{4}} l_{P(4)} \tag{2.29}$$

$$R_{\rm KK} \sim N^{\frac{7}{12}} l_{P(4)} \tag{2.30}$$

$$g_s \sim N^{-\frac{1}{4}}\,. \tag{2.31}$$

However in this regime, γ_1 (see eq. (2.9)) shrinks to zero, indicating that (massive) type IIA is not the correct description for this case. The above scalings might thus not hold in this scaling limit.

3. Approximate double T-duality

The DeWolfe et al. model is formulated in massive type IIA SUGRA. This theory does not have a perturbative world sheet expansion and quantization of the F_0 is problematic. The second difficulty is that the model also contains an orientifold which is a singular object when described in type IIA SUGRA [11]. To study the model, we will first apply two T-dualities, ignoring back reaction of the orientifold. These will bring us to non-massive type IIA. We will address the second problem by inserting the orientifold in the dualized configuration. The T-duality transformations have the additional benefit that the H_3 flux vanishes. The original H_3 flux turns into a geometric flux showing up as twists in the metric.

3.1 Approximate character of the T-dualities

Let us first point out that applying a double T-duality on a configuration with fluxes results in general in a non-geometric compactification [12]. However, the T-dualities we will perform are chosen such that we do not violate the condition ensuring that we remain in the domain of geometric compactification [13].

We will first perform a T-duality on the x_1-direction, followed by a T-duality in the x_2-direction. The T-duality transformations will only be valid in an approximate sense:

- The loops on the T^6 defined by the x_1 and x_2-directions are contractible on the fixed points of the $\mathbb{Z}_2 \times \mathbb{Z}_3 \times \mathbb{Z}_3$ identifications. We thus do not have an S^1-isometry required for an exact T-duality.

- In addition, we will work in the approximation where $F_2 = 0$. As discussed in section 2.3, this flux does not satisfy the Bianchi condition. We thus expect that the Bianchi identity after T-dualities will not be satisfied either. The F_2 flux is sourced by the orientifold and by the flux term $F_0 H_3$. In the T-duality computation we will keep track of the fluxes F_0, H_3 and the cycle on which the orientifold is wrapped. This information will be helpful to correct the T-dualized solution.

- Notice that we can expect the correct F_2 in the original setup to depend on all coordinates x_i, since we expect that close to the orientifold the F_2 flux resembles the F_2 flux of an orientifold in flat space. Because of the identifications the orientifold is extended along all coordinate directions (see figure 1). This implies that the F_2 flux breaks the S^1-isometry in the coordinate directions. Thus, if we would include the back reaction of the orientifold, which sources the F_2-flux, we would not be able to T-dualize.

There are two related ways to view our approximate T-dualities. So far we have emphasized the first, which is the interpretation of T-duality mapping solutions of Type IIA supergravity into other solutions. In our case it takes solutions of massive Type IIA into solutions of ordinary Type IIA, because the duality eliminates the F_0 flux. This duality is only approximate and in order to perform it we must ignore the orientifold (or at least its back reaction).

Alternatively, we can start from the orbifold conformal field theory of DeWolfe et al. Turning on fluxes corresponds to deformations of the background in the direction of certain vertex operators, and the orientifold corresponds to modding out the CFT by one of its symmetry operations. We can perform an exact T-duality on the CFT (just a change of variables) and try to understand which vertex operators must be turned on in the T-dual language.[3] Similarly we can mod out by the T-dual symmetry operation. The result of this operation is a CFT just as mysterious as the one, one might have tried to write down in the original picture. However to leading order in the string tension expansion, it leads to a new set of equations of motion, to which we may try to find a solution. The reader may choose whichever interpretation of our procedure (s)he finds most convincing. We do not pretend that we have presented a rigorous argument for either approach.

At any rate, as a consequence of the approximations, we can expect the solution after T-dualities to contain inconsistencies. By imposing the equations of motion, the Bianchi conditions and supersymmetry conditions on the dualized configuration, we hope to find a tractable version of the DeWolfe et al. solutions, with a well defined large N expansion.

3.2 Orientifold projection in a twisted torus

The H_3-flux in the DeWolfe et al. paper leads to twisting in the geometry after T-duality. In this section we study how an orientifold fixed plane behaves under T-duality when a non-trivial H_3-flux background is turned on.

[3]The real problem here is that since fluxes are discrete, the sought for CFT is not a small perturbation of the original orbifold.

Since the H_3-flux (2.2) and the orientifold (2.17), have several components along different x_i-directions, we get several twisting terms in the metric after T-duality in the x_i coordinate system. The T-duality action allows us to break this problem in several smaller problems by focusing on one term in the orientifold and one term of the H_3-flux. Let us work out the case where we focus on the term

$$H_3 \sim dx_1 \wedge dx_4 \wedge dx_6 + \dots \tag{3.1}$$

and where the orientifold fixed plane is wrapped on the compact 3-cycle

$$\mathrm{O6} : dx_1 \wedge dx_3 \wedge dx_5 + \dots \tag{3.2}$$

in the Calabi-Yau manifold. Let us rename and rescale, x_1, x_4, x_6 as θ, Y, Z. We can now focus on the 3-torus T^3 with H_3-flux [13]:

$$ds_3^2 = d\theta^2 + dY^2 + dZ^2 \tag{3.3}$$

$$H_3 = d\theta \wedge dY \wedge dZ\,, \tag{3.4}$$

where we take θ, Y and Z to have periodicities 2π. The location of the orientifold in this T^3 subspace of the compact manifold is determined by the fixed plane of the symmetry,

$$\theta \to -\theta \tag{3.5}$$

$$Y \to Y \tag{3.6}$$

$$Z \to Z\,. \tag{3.7}$$

The bosonic part of the worldsheet action which encodes the dynamics on the T^3 is given by ($d^2z = d\sigma^1 d\sigma^2$):

$$S = \frac{1}{2\pi\alpha'}\int d^2z\{\partial\theta\bar\partial\theta + \partial Y\bar\partial Y + \partial Z\bar\partial Z + Y\partial\theta\bar\partial Z - Y\partial Z\bar\partial\theta\}\,. \tag{3.8}$$

This action is invariant under the periodic identifications of the target space coordinates since the periodic identification, $Y \to Y + 2\pi$, only contributes a total derivative. The action is also invariant under the discrete orientifold symmetry $\sigma\Omega_\theta$:

$$\theta(z,\bar z) \to -\theta(\bar z, z) \tag{3.9}$$

$$Y(z,\bar z) \to Y(\bar z, z) \tag{3.10}$$

$$Z(z,\bar z) \to Z(\bar z, z)\,. \tag{3.11}$$

Now we can perform a T-duality in the θ-direction by gauging the U(1)-isometry along that direction [14]. The new action reads,

$$S = \frac{1}{2\pi\alpha'}\int d^2z\{(\partial\theta + A)(\bar\partial\theta + \bar A) + \partial Y\bar\partial Y + \partial Z\bar\partial Z \tag{3.12}$$

$$+ Y(\partial\theta + A)\bar\partial Z - Y\partial Z(\bar\partial\theta + \bar A) + \tilde\theta F\}\,, \tag{3.13}$$

with $F = \partial\bar A - \bar\partial A$. The field $\tilde\theta$, is a Lagrange multiplier with period 2π. Integrating out $\tilde\theta$ gives the original action.

The new action is only invariant under the periodicity $Y \to Y + 2\pi$, if we take $\tilde{\theta} \to \tilde{\theta} - 2\pi Z$, simultaneously. We can also extend the action of the orientifold symmetry by requiring that the new action is invariant under the extended orientifold symmetry. The orientifold symmetry, $\sigma\Omega_\theta$, becomes:

$$\theta(z,\bar{z}) \to -\theta(\bar{z},z) \qquad Y(z,\bar{z}) \to Y(\bar{z},z) \tag{3.14}$$

$$A(z,\bar{z}) \to -A(\bar{z},z) \qquad Z(z,\bar{z}) \to Z(\bar{z},z) \tag{3.15}$$

$$\bar{A}(z,\bar{z}) \to -\bar{A}(\bar{z},z) \qquad F(z,\bar{z}) \to F(\bar{z},z) \tag{3.16}$$

$$\tilde{\theta}(z,\bar{z}) \to \tilde{\theta}(\bar{z},z)\,. \tag{3.17}$$

Fixing the gauge with the condition $\theta = 0$ and integrating out the fields A and $\bar{A}$ gives the dual action:

$$S = \frac{1}{2\pi\alpha'}\int d^2z\{(\partial\tilde{\theta} + Y\partial Z)(\bar{\partial}\tilde{\theta} + Y\bar{\partial}Z) + \partial Y\bar{\partial}Y + \partial Z\bar{\partial}Z\}\,. \tag{3.18}$$

Translating this to the target space gives,

$$ds_3^2 = (d\tilde{\theta} + YdZ)^2 + dY^2 + dZ^2 \tag{3.19}$$

$$H_3 = 0\,, \tag{3.20}$$

with the identifications,

$$\tilde{\theta} \to \tilde{\theta} + 2\pi \tag{3.21}$$

$$Y \to Y + 2\pi \quad \text{and} \quad \tilde{\theta} \to \tilde{\theta} - 2\pi Z \tag{3.22}$$

$$Z \to Z + 2\pi\,. \tag{3.23}$$

The metric is thus a circle bundle over a torus. The non-trivial identifications indicate that the coordinate $\tilde{\theta}$, along the fibre is not globally well-defined. Let us introduce the one form Θ, which is globally defined by $d\Theta = dY \wedge dZ$ and gives locally $\Theta = d\tilde{\theta} + YdZ$. The action of the orientifold symmetry now reads:

$$\Theta \to \Theta \tag{3.24}$$

$$Y \to Y \tag{3.25}$$

$$Z \to Z\,. \tag{3.26}$$

This is, for the example we considered, the orientifold wraps both the fibre and the torus base space after T-duality. The Poincaré dual form of the cycle on which the orientifold is wrapped becomes:

$$\mathrm{O7}: dx_3 \wedge dx_5 + \ldots\,. \tag{3.27}$$

We can repeat this exercise for different combinations of terms in the H_3-flux and orientifold cycle. We find that the orientifold either wraps the fibre of the twisted torus, $\Theta \to \Theta$, or reflects the fibre $\Theta \to -\Theta$. In the T-duality computation of the DeWolfe et al. solution, we will follow the orientifold by keeping track of the Poincaré dual form of the cycle on which the orientifold is wrapped. The above discussion shows that this is a consistent treatment of the orientifold.

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3.3 Doubly dualized background

In appendix A, we review the action of T-duality on a background of SUGRA. In appendix B, we work out the double T-duality transformation of the DeWolfe et al. model. The result reads:

$$
\begin{aligned}
ds^2 &= \frac{4\pi^2\alpha'}{\gamma_1}\left(9^{\frac{2}{3}}\,\Theta_1^2 + 4^2\,3^{-\frac{2}{3}}\,\Theta_2^2\right) + \gamma_2(dx_3^2+dx_4^2) + \gamma_3(dx_5^2+dx_6^2) \\
&\quad + ds^2_{\mathrm{AdS}_4} && (3.28)\\
H_3 &= 0 && (3.29)\\
e^{\varphi} &= \frac{1}{4}|h_3|\sqrt[4]{\frac{3^3 5}{|f_0 f_4^1 f_4^2 f_4^3|}} && (3.30)\\
F_4 &= 4(2\pi\sqrt{\alpha'})^3\sqrt[3]{3} f_4^1 \frac{1}{\gamma_2\gamma_3}\mathrm{vol}_4 && (3.31)\\
F_2 &= -4(2\pi\sqrt{\alpha'})^3\frac{\sqrt[3]{3}}{\gamma_1}(f_4^2\,dx_5\wedge dx_6 + f_4^3\,dx_3\wedge dx_4) \\
&\quad + f_0\frac{2\pi\sqrt{\alpha'}}{\gamma_1}\,4\cdot 3^{\frac{1}{3}}\,\Theta_1\wedge\Theta_2 && (3.32)\\
F_0 &= 0 && (3.33)\\
\frac{1}{2\kappa^2_{10\tilde{A}}} &= \frac{1}{2\kappa^2_{10A}}\frac{\gamma_1^2}{(4\pi^2\alpha')^2\,4\cdot 3^{\frac{1}{3}}} && (3.34)\\
\Theta_1 &= 2\pi\sqrt{\alpha'}dx_1 + 2\pi\sqrt{\alpha'}h_3\frac{\sqrt[4]{3}\sqrt{2}}{9^{\frac{1}{3}}}(x_3dx_5 - x_4dx_6) && (3.35)\\
\Theta_2 &= 2\pi\sqrt{\alpha'}dx_2 + 2\pi\sqrt{\alpha'}h_3\frac{\sqrt[4]{3}\sqrt{2}}{4\cdot 3^{-\frac{1}{3}}}(-x_3dx_6 - x_4dx_5)\,, && (3.36)
\end{aligned}
$$

where $x_1 \in [0, 9^{-1/6}]$ and $x_2 \in [0, 2^{-1}\,3^{1/6}]$.

The original orientifold splits into an O5- and O7-plane after the first T-duality (see appendix B). The second T-duality recombines those two orientifold planes to give an O6-plane wrapped on the Poincaré dual of the $\tilde{\alpha}_0$-cycle:

$$
\begin{aligned}
\mathrm{O6}\ :\ \sqrt[4]{3}\sqrt{2}\Bigg(&+\frac{2\cdot 3^{-\frac{1}{6}}\Theta_2}{2\pi\sqrt{\alpha'}\,9^{\frac{1}{6}}}\wedge(dx_3\wedge dx_5 - dx_4\wedge dx_6) && (3.37)\\
&+\frac{9^{\frac{1}{6}}\Theta_1}{2\pi\sqrt{\alpha'}\,2\cdot 3^{-\frac{1}{6}}}\wedge(dx_4\wedge dx_5 + dx_3\wedge dx_6)\Bigg)\\
=\ &\tilde{\beta}_0\,. && (3.38)
\end{aligned}
$$

4. The Bianchi identity after the double T-duality

As mentioned earlier, we expect the dualized solution (3.28)–(3.36) to contain inconsistencies. We will do the full analysis of the consistency conditions later. Here we will focus on the Bianchi condition. Taking the F_2 flux from the dualized solution we compute:

$$
\begin{aligned}
dF_2 &= f_0 \frac{2\pi\sqrt{\alpha'}}{\gamma_1} 4 \cdot 3^{\frac{1}{3}} (d\Theta_1 \wedge \Theta_2 - \Theta_1 \wedge d\Theta_2) &(4.1)\\
&= f_0 h_3 \frac{(2\pi\sqrt{\alpha'})^3}{\gamma_1} 2 \cdot 3^{\frac{1}{6}} \tilde{\beta}_0 \,. &(4.2)
\end{aligned}
$$

This 3-form is everywhere non-zero.

On the other hand, since the configuration after T-dualities is a solution of massless type IIA string theory, with the orientifold as only source for F_2, we expect the Bianchi identity to read:

$$
\frac{1}{2\kappa^2_{10\tilde{A}}} dF_2 = \mu_6 \delta_{\mathrm{O6}} = \mu_6 \delta(\tilde{\beta}_0) \,. \tag{4.3}
$$

The distributional 3-form dF_2, is thus localized on the orientifold plane, which lies on the Poincaré dual of the 3-form $\tilde{\beta}_0$. This is clearly at odds with (4.2). This inconsistency was not unexpected as mentioned earlier.

We will now modify the dualized background such that it satisfies the Bianchi condition. From equation (4.3) we get:

$$
dF_2 = -2 \frac{(2\pi\sqrt{\alpha'})^3}{\gamma_1} 2 \cdot 3^{\frac{1}{6}} \delta(\tilde{\beta}_0) \,. \tag{4.4}
$$

Integration over the $\tilde{\beta}_0$-cycle gives,

$$
\int_{\tilde{\beta}_0} dF_2 = \int_{\mathrm{vol}_6} dF_2 \wedge *_6 \tilde{\beta}_0 = -2 \frac{(2\pi\sqrt{\alpha'})^3}{\gamma_1} 2 \cdot 3^{\frac{1}{6}} \,, \tag{4.5}
$$

or, after partial integration,

$$
2 \cdot 3^{\frac{1}{6}} \frac{\gamma_1}{(2\pi\sqrt{\alpha'})^3} h_3 \int_{\mathrm{vol}_6} F_2 \wedge (dx_3 \wedge dx_4 \wedge dx_5 \wedge dx_6) = -2 \,. \tag{4.6}
$$

In the original DeWolfe et al. solution, the flux from the orientifold was absorbed by the $F_0 H_3$ term in the Bianchi identity. This leads to the constraint $f_0 h_3 = -2$. The above derivation shows how the twisted geometry, with the non-closed 3-form $*_6\tilde{\beta}_0$, absorbs the orientifold flux without the $F_0 H_3$ flux term.

The integrated Bianchi condition also gives us some information on the correct F_2 flux. It should contain a (distributional) term proportional to $f_0 \Theta_1 \wedge \Theta_2$. We also learn that as $N \to \infty$, the F_2 flux has to decrease. From now on we will ignore the term,

$$
f_0 \frac{2\pi\sqrt{\alpha'}}{\gamma_1} 4 \cdot 3^{\frac{1}{3}} \Theta_1 \wedge \Theta_2 \,, \tag{4.7}
$$

in (3.32). Instead we will include a term $F_{2,\,\mathrm{O6}}$ which satisfies (4.4).

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5. Lift to M-theory

5.1 Entropy computation and motivation for an M-theory interpretation

Let us return to the original DeWolfe et al. solution. It is supposed to be an AdS_4 spacetime, Maldacena dual to a $2+1$ dimensional CFT. Following [15] we can calculate the entropy of this system,

$$S_{\mathrm{BH}} = \frac{A^{\mathrm{Einst}}_{\mathrm{horizon}}}{4G_N^{4\mathrm{d}}} \tag{5.1}$$

$$= \pi \left(2MR_{\mathrm{AdS}}\right)^{\frac{2}{3}} \left(\frac{160 \cdot 2^{\frac{7}{8}} 3^{\frac{3}{4}} 5^{\frac{1}{4}} \pi}{27} \frac{1}{|f_0|^{\frac{5}{4}} |h_3|^2} |f_4^1 f_4^2 f_4^3|^{\frac{3}{4}} \right)^{\frac{2}{3}} , \tag{5.2}$$

where M is the mass of the black hole.

Let us now consider the different scalings of the four form flux as discussed in section 2.4. For the regime $f_4^1 = f_4^2 = f_4^3 = N$, we find that

$$S_{\mathrm{BH}} \sim N^{\frac{3}{2}} . \tag{5.3}$$

This gives us the entropy as a function of the energy of the black hole. Using standard CFT thermodynamics, this implies a scaling

$$S_{\mathrm{BH}} \sim N^{\frac{9}{2}} . \tag{5.4}$$

as a function of the temperature.[4] We do not know of a conformal field theory with this kind of scaling.

If we take the other scaling regime where $f_4^1 \sim N$, while the other two four form fluxes are held fixed, then we find an entropy scaling like $N^{3/2}$ at fixed temperature, in the large N limit. We know that the entropy of the CFT describing a stack of N M2-branes scales in precisely this manner [17]. This computation thus seems to indicate that we should look for an M-theory interpretation of the special cases of the DeWolfe et al. background, with one large four form flux and the others of order one. We note that repeating the entropy computation in the doubly dualized background gives exactly the same answer as (5.2).

Indeed, we found that in our double T-dual solution, there was four form flux in the AdS_4 directions, corresponding to of order N D2-branes at the end of the universe. Note that this flux comes only from f_4^1, the flux we have chosen to be large in order to get M2-branes entropy scaling. Our entropy calculation suggests that these D-branes are behaving like M2-branes. This could be explained, if the IIA string coupling on the compact manifold is strong in the region where the M2-branes are localized.

There is a second reason indicating that we should look for an M-theory setting of the problem. The orientifold is a singular object in 10D SUGRA. The M-theory lift of an orientifold in flat space is the Atiyah-Hitchin manifold [18, 19]. In 11 dimensions, we have thus a non-singular, completely geometric description. The orientifold is singular in the Type IIA limit, because the string coupling is always large in the core of the Atiyah-Hitchin

[4]This computation was done independently in [16]

manifold, no matter what its value is at infinity. Since the orientifold locus includes the AdS_4 directions, the D2-branes in our T-dual configuration are sitting in a strong coupling region. This explains why they behave like M2-branes.

Such a picture is inconsistent with a weak coupling string theory interpretation of the DeWolfe et al. configurations, with only one large flux. We note that these observations are valid in the region where f_4^1 is large, and the other two four form fluxes are non-zero, and may be large or small. DeWolfe et al. only claimed to have a weakly coupled four dimensional compactification in the region where all fluxes are large. In our T-dual picture, even this regime has a large number of branes sitting near the orientifold. Our next thought was that there might be an M-theory interpretation with N M2-branes embedded in a smooth manifold. We will see that this is possible only for a single large flux with the other fluxes fixed and non-zero. Although the calculations of DeWolfe et al. still indicate a weakly coupled four dimensional compactification in this limit, certain cycles of the compact manifold shrink to zero for large N. These authors do not claim to have control over the regime that we claim has a smooth M-theory limit with comparable AdS_4 and M_7 radii.

5.2 Lift to M-theory

Given a massive type IIA solution (without H_3 flux), C. M. Hull constructed a procedure to lift the solution to M-theory [20]. This process consists roughly of a T-duality to type IIB and then a lift via F-theory to M-theory.[5] As discussed earlier, if we T-dualize the DeWolfe et al. background once, some H_3 flux remains which complicates the lift to M-theory. Therefore, we will follow the slightly different track of T-dualizing twice and then using the strong-weak correspondence between type IIA string theory and M-theory to lift the configuration to 11D [21].

In the 10D theories the orientifold is a singular object which we included by keeping track of the cycle on which it was wrapped and via its source term in the Bianchi identity. As mentioned earlier, in M-theory this singular object translates into a non-singular geometric object. Its explicit form is only known in the case of an orientifold in flat space [23]. Our strategy will be to first construct a naive lift ignoring the orientifold. Appendix C reviews the formulas to lift a non-singular type IIA SUGRA background to M-theory. However, omitting the orientifold will introduce inconsistencies. In a second step, we will impose the consistency conditions and try to modify the naive lift.

Constructing the naive lift to M-theory of the dualized background gives, with $L_M =$

[5] There are complications on the quantum level with this construction [22]. Our analysis has been on the classical level.

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$2\pi\frac{\kappa_{10\tilde{A}}}{\sqrt{\pi}(2\pi\sqrt{\alpha'})^3}$:

$$\begin{aligned}
ds^2 &= R_M^2\Theta_M^2+\tilde{\gamma}_{11}\Theta_1^2+\tilde{\gamma}_{12}\Theta_2^2+\tilde{\gamma}_2(dx_3^2+dx_4^2)+\tilde{\gamma}_3(dx_5^2+dx_6^2)+ds^2_{\mathrm{AdS}_4} && (5.5)\\
G_4 &= 6m\,\mathrm{vol}_4 && (5.6)\\
\Theta_1 &= dx_1+h_3\frac{\sqrt[4]{3}\sqrt{2}}{9^{\frac{1}{3}}}(x_3dx_5-x_4dx_6) && (5.7)\\
\Theta_2 &= dx_2+h_3\frac{\sqrt[4]{3}\sqrt{2}}{4\cdot 3^{-\frac{1}{3}}}(-x_3dx_6-x_4dx_5) && (5.8)\\
\Theta_M &= dx_M+A_1 && (5.9)\\
dA_1 &= -2\cdot 3^{\frac{1}{6}}(f_4^2dx_5\wedge dx_6+f_4^3dx_3\wedge dx_4)+\tilde{F}_{2,\,\mathrm{O6}} && (5.10)\\
\frac{1}{2\kappa_{11M}^2} &= \frac{1}{16\pi l_{P11}^9}=\frac{1}{2\kappa_{10A}^2}\frac{\gamma_1^2}{(4\pi^2\alpha')^2\,4\cdot 3^{\frac{1}{3}}}\frac{1}{L_M}\,. && (5.11)
\end{aligned}$$

where $\tilde{F}_{2,\,\mathrm{O6}}=F_{2,\,\mathrm{O6}}/L_M$ and with

$$\begin{aligned}
R_M &= \frac{3^{\frac{5}{6}}\pi^{\frac{2}{9}}}{2^{\frac{7}{9}}5^{\frac{1}{6}}}|f_0h_3^4|^{\frac{1}{6}}\frac{|f_4^1|^{\frac{1}{6}}}{|f_4^2f_4^3|^{\frac{1}{2}}}l_{P11} && (5.12)\\
\tilde{\gamma}_{11} &= \frac{2\cdot 2^{\frac{4}{9}}3^{\frac{5}{6}}\pi^{\frac{4}{9}}}{5^{\frac{1}{3}}}\frac{|f_0|^{\frac{1}{3}}}{|h_3|^{\frac{2}{3}}}|f_4^1|^{\frac{1}{3}}l_{P11}^2 && (5.13)\\
\tilde{\gamma}_{12} &= \frac{32\cdot 2^{\frac{4}{9}}3^{\frac{5}{6}}\pi^{\frac{4}{9}}}{9\cdot 5^{\frac{1}{3}}}\frac{|f_0|^{\frac{1}{3}}}{|h_3|^{\frac{2}{3}}}|f_4^1|^{\frac{1}{3}}l_{P11}^2 && (5.14)\\
\tilde{\gamma}_2 &= \frac{8\cdot 2^{\frac{4}{9}}3^{\frac{5}{6}}5^{\frac{2}{3}}\pi^{\frac{4}{9}}}{9}\frac{1}{|f_0h_3|^{\frac{2}{3}}}|f_4^1|^{\frac{1}{3}}f_4^3l_{P11}^2 && (5.15)\\
\tilde{\gamma}_3 &= \frac{8\cdot 2^{\frac{4}{9}}3^{\frac{5}{6}}5^{\frac{2}{3}}\pi^{\frac{4}{9}}}{9}\frac{1}{|f_0h_3|^{\frac{2}{3}}}|f_4^1|^{\frac{1}{3}}f_4^2l_{P11}^2 && (5.16)\\
R_{\mathrm{AdS}} &= \frac{8\cdot 2^{\frac{2}{9}}3^{\frac{5}{6}}5^{\frac{5}{6}}\pi^{\frac{2}{9}}}{9}\frac{1}{|f_0|^{\frac{5}{6}}|h_3|^{\frac{4}{3}}}|f_4^1|^{\frac{1}{6}}|f_4^2f_4^3|^{\frac{1}{2}}l_{P11} && (5.17)\\
m &= \frac{2^{\frac{7}{9}}3^{\frac{7}{6}}5^{\frac{1}{6}}}{160\pi^{\frac{2}{9}}}|f_0|^{\frac{5}{6}}|h_3|^{\frac{4}{3}}\frac{f_4^1}{|f_4^1|^{\frac{7}{6}}|f_4^2f_4^3|^{\frac{1}{2}}}\frac{1}{l_{P11}}\,. && (5.18)
\end{aligned}$$

Notice that we also rescaled Θ_1,Θ_2 by $1/(2\pi\sqrt{\alpha'})$ as compared to the previous sections.

6. Discussion of the naive M-theory lift

6.1 Condition on $F_{2,\,\mathrm{O6}}$

The term $L_M\tilde{F}_{2,\,\mathrm{O6}}=F_{2,\,\mathrm{O6}}$ in (5.10) has to satisfy the Bianchi identity (4.4). Integration as in (4.6) leads to the condition

$$4\cdot 3^{\frac{1}{3}}h_3\int_{\mathrm{vol}_6}\tilde{F}_{2,\,06}\wedge(dx_3\wedge dx_4\wedge dx_5\wedge dx_6)=-2\,. \qquad (6.1)$$

As discussed earlier, this constraint should lead to $f_0h_3=-2$.

6.2 Volume of the compact manifold

The volume of the compact 7 dimensional manifold is

$$\mathrm{vol}_7 = \frac{64 \cdot 2^{\frac{5}{9}} 3^{\frac{5}{6}} 5^{\frac{5}{6}} \pi^{\frac{14}{9}}}{27} \frac{1}{|f_0|^{\frac{5}{6}} |h_3|^{\frac{4}{3}}} |f_4^1|^{\frac{7}{6}} |f_4^2 f_4^3|^{\frac{1}{2}} l_{P11}^7 \,. \tag{6.2}$$

The Kaluza-Klein radius becomes

$$R_{\mathrm{KK}} = \sqrt[7]{\mathrm{vol}_7} \tag{6.3}$$

$$= \frac{2^{\frac{59}{63}} 3^{\frac{29}{42}} 5^{\frac{5}{42}} \pi^{\frac{2}{9}}}{3} \frac{1}{|f_0|^{\frac{5}{42}} |h_3|^{\frac{4}{21}}} |f_4^1|^{\frac{1}{6}} |f_4^2 f_4^3|^{\frac{1}{14}} l_{P11} \,. \tag{6.4}$$

6.3 The entropy

Let us express the entropy as a function of the energy of the black hole as in (5.2). The scaling part of the entropy of the configuration is given by,

$$S_{\mathrm{BH}} \sim \left(\frac{R_{\mathrm{AdS}}}{l_{P4}} \right)^{\frac{2}{3}} \tag{6.5}$$

$$\sim \left(\frac{160 \cdot 2^{\frac{7}{8}} 3^{\frac{3}{4}} 5^{\frac{1}{4}} \pi}{27} \frac{1}{|f_0|^{\frac{5}{4}} |h_3|^2} |f_4^1 f_4^2 f_4^3|^{\frac{3}{4}} \right)^{\frac{2}{3}} \,. \tag{6.6}$$

Comparing this to the scaling part of the entropy in the original setup (see equation (5.2)), we see that they match perfectly.

6.4 Scaling behavior

In the regime $f_4^1 = f_4^2 = f_4^3 = N$, the various parameters of the background scale as,

$$R_M \sim N^{-\frac{5}{6}} l_{P11} \tag{6.7}$$

$$R_{\mathrm{AdS}} \sim N^{\frac{7}{6}} l_{P11} \tag{6.8}$$

$$\mathrm{vol}_7 \sim N^{\frac{13}{6}} l_{P11}^7 \tag{6.9}$$

$$R_{\mathrm{KK}} \sim N^{\frac{13}{42}} l_{P11} \tag{6.10}$$

$$m \sim N^{-\frac{7}{6}} l_{P11}^{-1} \tag{6.11}$$

$$S_{\mathrm{BH}} \sim N^{\frac{3}{2}} \,, \tag{6.12}$$

where we express the scaling of the entropy as a function of the energy as in (5.2). We see that just as the original DeWolfe et al. solution, the M-theory configuration is effectively 4 dimensional, since R_{AdS} grows faster with N than R_{KK}. Note that the same analysis on the doubly dualized type IIA background teaches us that background is also effectively 4 dimensional in this particular scaling regime. The radii characterizing the solution grow as N increases, except the M-theory radius R_M, which decreases with growing N. We will discuss this property below.

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Taking a look at the other scaling $f_4^1 = N, f_4^2 = f_4^3 = O(1)$, we get,

$$R_M \sim N^{\frac{1}{6}} l_{P11} \tag{6.13}$$
$$R_{\text{AdS}} \sim N^{\frac{1}{6}} l_{P11} \tag{6.14}$$
$$\text{vol}_7 \sim N^{\frac{7}{6}} l_{P11}^7 \tag{6.15}$$
$$R_{\text{KK}} \sim N^{\frac{1}{6}} l_{P11} \tag{6.16}$$
$$m \sim N^{-\frac{1}{6}} l_{P11}^{-1} \tag{6.17}$$
$$S_{\text{BH}} \sim N^{\frac{1}{2}} . \tag{6.18}$$

Here we conclude that the AdS and the compact manifold grow at the same rate such that the compactification is not effectively four dimensional. On the other hand, in this case all the radii of the 11 dimensional solution grow with N making 11D SUGRA a valid approximation for large N. As previously mentioned, the scaling of the entropy as function of the temperature in this regime is $N^{3/2}$.

6.5 Checking the consistency conditions

6.5.1 M-theory equations of motion

From (C.1), we get the equation of motion for the metric:

$$\text{Ric}_{MN} = \frac{2}{4!}\left(G_{MPQR}\, G_N{}^{PQR} - \frac{1}{12} g_{MN}\, G_{PQRS}\, G^{PQRS}\right) . \tag{6.19}$$

Taking the indices M, N in the AdS space, this condition reduces to:

$$\frac{1}{R_{\text{AdS}}^2} = 4m^2 . \tag{6.20}$$

This condition is satisfied as we can verify from (5.18).

The equation of motion for the compact part of the metric, $g_{mn}^{(7)}$, becomes:

$$\text{Ric}_{mn} = 6m^2 g_{mn}^{(7)} . \tag{6.21}$$

This implies that the compact 7-manifold is an Einstein manifold. As is common in similar cases, this condition will be satisfied if the supersymmetry condition is satisfied.

We can verify that the equation of motion and Bianchi condition on G_4,

$$d * G_4 + \frac{1}{2} G_4 \wedge G_4 = 0 \tag{6.22}$$
$$dG_4 = 0 , \tag{6.23}$$

are satisfied.

6.5.2 Supersymmetry conditions

The original background was an $\mathcal{N} = 1$ compactification in four dimensions. From [24], we learn that the supersymmetry requirement on the M-theory lift, $\text{AdS}_4 \times M_7$, is that M_7

has weak G_2 holonomy. Weak G_2 holonomy of a 7-manifold is defined by the condition that there exists a 3-form ϕ_3 and a real number m such that,

$$d\phi_3 = 4m *_7 \phi_3 \,. \tag{6.24}$$

From this condition one can derive the equation of motion (6.21) [25]. We conclude that if the supersymmetry conditions are obeyed then the naive M-theory lift is fully consistent.

When $N \to \infty$, $m \to 0$ and the supersymmetry condition on the compact manifold simplifies to G_2 holonomy:

$$d\phi_3 = 0 \tag{6.25}$$

$$d *_7 \phi_3 = 0 \,. \tag{6.26}$$

The 4 dimensional analysis in [1] led to stricter conditions on the signs of the F_4 flux parameters f_4^1, f_4^2 and f_4^3:[6]

$$\mathrm{sign}(f_0 f_4^1 f_4^2 f_4^3) < 0 \tag{6.27}$$

$$\mathrm{sign}(f_4^1) = \mathrm{sign}(f_4^2) = \mathrm{sign}(f_4^3) \,. \tag{6.28}$$

Backgrounds violating the above condition are believed to be stable but non-supersymmetric solutions [1]. We can thus expect that the above conditions will follow from the weak G_2 holonomy condition.

If we check the weak G_2 holonomy condition for the naive lift, we find that it does not satisfy the conditions. We included the implicitly determined flux $\tilde{F}_{2,\,\mathrm{O6}}$ which is sourced by the orientifold, while we did not include the Atiyah-Hitchin like geometry from the orientifold. As the coupling constant flows from type IIA to M-theory, we expect the singular orientifold to get some thickness, modifying the geometry in the region close to the orientifold. We thus expect that the naive lift is only an approximation for the geometry far away from the orientifold.

We did not succeed in finding an explicit solution to (6.24) in the regime where $f_4^1 = N, f_4^2 = f_4^3 = O(1)$, but neither have we found any obstruction to the existence of a metric of weak G_2 holonomy with the scaling properties and behavior near the M2-brane locus that we were led to. We believe that in the limit of a single large flux there is a systematic M-theory expansion. The background is $AdS_4 \times M_7$, with M_7 a manifold of weak G_2 holonomy. The anti de Sitter and M_7 radii are comparable. For other configurations of large flux we believe that the M-theory picture is only valid locally, in the vicinity of the orientifold, but that this region is large and cannot be ignored for large N. The string coupling *does* go to zero over another large region of the manifold.

6.6 Interpretation as a stack of M2-branes

The entropy argument of section 5.1 indicated that for a certain flux configuration we could expect the DeWolfe et al. solution to be the near horizon of a stack of M2-branes. The

[6]Equation (6.28) follows from the Kähler cone conditions for the background. There are additional Kähler cone conditions for the blow ups of the singularities. We do not consider those conditions here since our strategy was to ignore the singularities in the first step.

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$\mathrm{AdS}_4 \times M_7$ background with a weak G_2 holonomy condition on M_7, as discussed in the previous section, is in [24] indeed interpreted as the near horizon limit of M2-branes.

The background of a stack of N M2-branes at the tip of a cone, is given by

$$ds^2 = H^{-\frac{2}{3}} ds_3^2 + H^{\frac{1}{3}} ds_8^2 \tag{6.29}$$
$$G_4 = \mathrm{vol}_3 \wedge dH^{-1}\,, \tag{6.30}$$

with ds_3^2 and vol_3 the Minkowski metric and worldvolume of the M2-branes and $ds_8^2 = du^2 + u^2 ds_7^2$ the metric of the cone in the directions transverse to the M2-branes. The function H is given by

$$H = 1 + \frac{a^6}{u^6}\,, \tag{6.31}$$

and a is determined by the number of M2-branes [26]:

$$a^6 = N \frac{\kappa_{11M}^2 T_3}{3\Omega_7} = N \frac{\kappa_{11M}^2}{3a^{-7}\mathrm{vol}_7} \left(\frac{4\pi^2}{2\kappa_{11M}^2}\right)^{\frac{1}{3}}\,, \tag{6.32}$$

with vol_7 the volume form on ds_7^2. The near horizon limit of this background becomes (after a coordinate transformation $r = 2u^2/a$):

$$ds^2 = \frac{r^2}{4a^2}(-dt^2 + dy_1^2 + dy_2^2) + \frac{a^2}{4r^2} dr^2 + a^2 ds_7^2 \tag{6.33}$$
$$G_4 = \frac{6}{a}\mathrm{vol}_4\,, \tag{6.34}$$

where vol_4 is the volume form on the AdS_4 space which has $R_{\mathrm{AdS}} = a/2$. Comparing this to the M-theory lift of the DeWolfe et al. solution (5.5), we find that $m = 1/a$ and using (6.32), we compute that the number of M2-branes is given by:

$$N = |f_4^1|\,. \tag{6.35}$$

We know that the entropy (as a function of energy) of the CFT corresponding to a stack of M2-branes scales as $N^{1/2} = |f_4^1|^{1/2}$. We can compare this to the scaling of the entropy of the M-theory lift (6.6), $|f_4^1 f_4^2 f_4^3|^{\frac{1}{2}}$. In the regime where $f_4^1 = N$, $f_4^2 = f_4^3 = O(1)$, the scaling of the entropy is exactly the same. We can thus interpret the M-theory lift of the DeWolfe et al. solution in that regime as a stack of N M2-branes at the tip of a cone. We do not have a deep understanding of the more generic regime, where all four form fluxes are large, nor the regime where two are large and one is small. We note again that De Wolfe et al. only claimed to have a systematic weak coupling and low energy expansion when all fluxes are large.

6.7 Validity of 11 dimensional supergravity in the generic regime

Let us consider the regime $f_4^1 = f_4^2 = f_4^3 = N$. The M-theory radius of the naive lift (6.7) decreases as N grows, indicating that the 11 dimensional supergravity approximation cannot be trusted, since the curvature of the background becomes too large and corrections to 11D SUGRA will be important. We find that $N < 3$ for $R_M > l_{P11}$. However, we also see that we need $N > 1$ for $\sqrt{\tilde{\gamma}_{11}} > l_{P11}$. We see that supergravity is only valid in a certain small range of values for N.

The above reasoning is entirely based on our naive lift. Including the orientifold in the geometry changes the situation close to the orientifold. For an orientifold embedded in flat space, the dilaton increases the closer you get to the orientifold. This corresponds to a larger M-theory radius R_M. We can expect the same behavior in our configuration: including the correct geometry coming from the orientifold will give an M-theory radius which is larger than our naive estimate.

The geometry of the 11D SUGRA solution that incorporates the orientifold, can be thought of as an interpolation between the region close to the orientifold (bolt-geometry) [23] and the region far away from the orientifold (naive lift). The twisted tori in the region away from the orientifold come from the $F_0 H_3$ term in the original Bianchi identity, while (part of) the twist in the M-theory direction corresponds to the F_2 flux sourced by the orientifold.

Let us now focus on large N. The size of the compact manifold is large, so there are points located far away from the orientifold. We expect this region far away from the orientifold to resemble our naive lift. Further yet from the orientifold we enter into a weak coupling region. This is the region where the original argument of DeWolfe et al. operates. In the large flux limit there is a large region where it fails. To see this, note that our T-dual Type IIA configuration has a flux in the AdS_4 directions, consistent with N D2-branes lying in the orientifold locus. The dilaton in such a D2-brane background has the form

$$e^{\varphi} = \left(1 + \frac{c_2 g_s N l_s^5}{r^5}\right)^{1/4} , \tag{6.36}$$

with c_2 a numerical constant. Plugging in the DeWolfe et al. value for the coupling at infinity, we find that the coupling gets large at a distance of order $N^{1/20} l_s$ from the stack of D2-branes. Thus, the weak coupling approximation breaks down over a parametrically large region of the manifold as $N \to \infty$.

If we make the plausible assumption that an 11D description is valid near the orientifold, we find that the local radius of curvature in the presence of the branes is of order the AdS radius. We connect the weak coupling geometry to a seven dimensional patch whose size is of order $N^{1/20} l_s \sim N^{13/60} l_{P11}$ (see figure 2), and whose geometry is that of the manifold of weak G_2 holonomy we described above. The radius of curvature of that geometry is of order the AdS radius, and thus, much larger than the size of the patch. There now seem to be two possibilities for a geometrical description of what is going on in the large N limit.[7] In the first, the patch is essentially flat (see figure 2a). Alternatively, the whole geometry could mushroom out to a large seven dimensional patch with weak G_2 holonomy and size of order the AdS radius (see figure 2b). We believe that neither the methods of DeWolfe et al. nor our own, are powerful enough to distinguish between these

[7]We emphasize that since we have *no* complete approximation scheme for this regime, there is no argument that *any* geometrical picture is valid.

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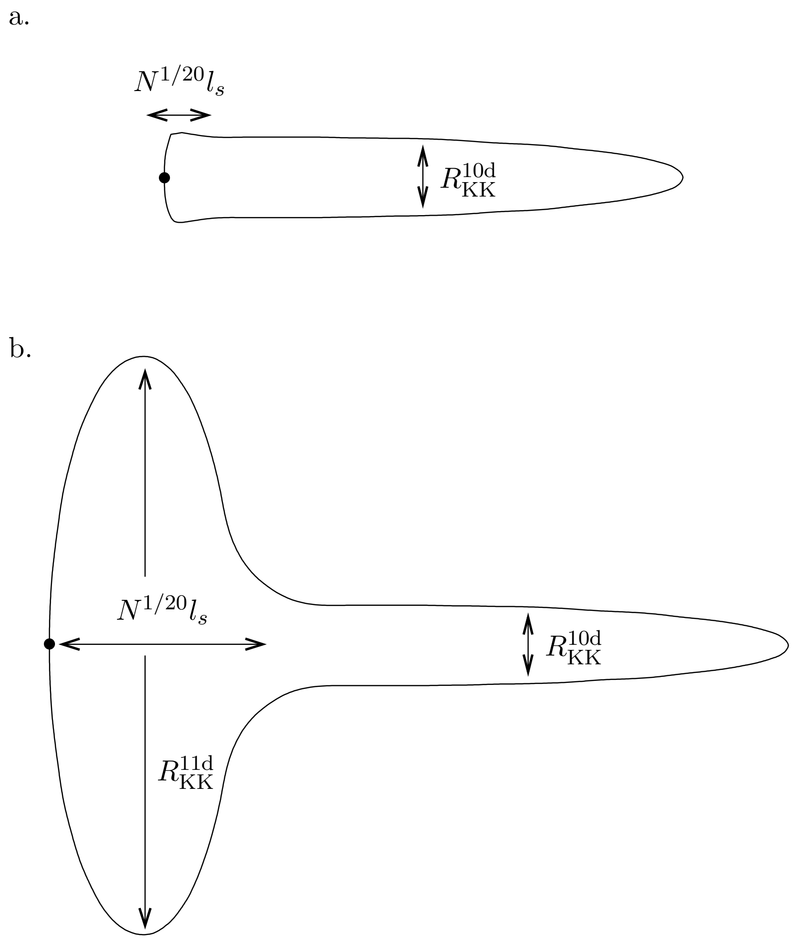

Figure 2: Two possible scenarios. The right hand side of each drawing represents the region where (massive) type IIA is the correct description and where the radius of curvature of the compact manifold, $R_{\mathrm{KK}}^{10\mathrm{d}}$, scales with N at a slower rate than R_{AdS}. The left hand side is the region close to the D2/orientifold locus (indicated with the dot). This region has a size that scales as $N^{1/20}l_s$. The compact manifold radii in this region, $R_{\mathrm{KK}}^{11\mathrm{d}}$, scale as fast as R_{AdS} scales with N, resulting in a flat patch (case a) or a mushroom cap (case b).

two alternatives. In the second alternative there would be KK excitations with a mass of order the inverse AdS radius, and the compactification would not be four dimensional.

7. Conclusions and speculation

We believe that we have provided ground for suspecting that the massive IIA description of the DeWolfe et al. background does not provide a systematic low energy expansion due to the back reaction of the orientifold. The doubly dualized description still has the same problem. However, this low energy effective description has the advantage that the F_0 and H_3 fluxes are absent.

Regime $f_4^1 = N$, $f_4^2 = f_4^3 = O(1)$: The scaling of the entropy indicates that there might be a correct expansion using 11D SUGRA in this regime. We constructed a naive lift to 11 dimensions. We gave arguments that a 7-manifold of weak G_2 holonomy exists and that N M2-branes at an approximately Atiyah-Hitchin locus on this manifold might give a description of the physics of these compactifications. We reiterate that this is not a regime where DeWolfe et al. claimed to have a controlled expansion.

We thus claim that in the regime where f_4^1 is large, and the other four form fluxes are of order 1, there should be a valid 11D SUGRA approximation to the DeWolfe et al. models. This would be the near horizon limit of the configuration of f_4^1 M2-branes at the tip of a cone over a seven manifold M_7 of weak G_2 holonomy. The linear size of M_7 scales in the same way as the AdS_4 radius. The exact description of this regime would be a $2+1$ dimensional CFT with fixed temperature entropy of order $(f_4^1)^{3/2}$. It should be possible to find it as the endpoint of the RG flow along a relevant perturbation of the CFT of M2-branes in flat space, which breaks the symmetry down to minimal $2+1$ dimensional SUSY. The supergravity solution would enable one to compute dimensions of low dimension operators at this fixed point. However, since the SUSY algebra is so small, there might not be any checks of these computations at the UV fixed point.

There is no sense in which this model is well approximated by weakly coupled string theory. In addition, the compactification is not approximately four dimensional. The AdS and M_7 radii are comparable. If our picture is the correct one, the failure of the weak coupling analysis should be attributed to the naive treatment of the orientifold. In M-theory, the center of the orientifold is a locus of strong IIA coupling. In these compactifications, for large f_1^4 (in the T-duality frame we have chosen), a large number of M2 branes sit at this locus, and their back reaction completely changes the weak coupling geometrical picture. Of course, the limit of a single large flux was not controllable in the picture of DeWolfe et al.. Nonetheless it is striking that a single shrinking cycle (from their point of view) can actually lead to a completely different picture of the geometry, and of the strength of the coupling.

Generic regime $f_4^1 = f_4^2 = f_4^3 = N$: This regime is more mysterious. The fixed temperature entropy of the CFT scales like $N^{9/2}$. We would like to propose a heuristic explanation of this scaling law, but we warn the reader that many aspects of this proposal are obscure. Klebanov and Tseytlin proposed an explanation [27] of the N^3 scaling of the $(2,0)$ CFT that describes M5-branes, in terms of partially BPS states of membranes in a pair of pants configuration with boundaries on three different 5-branes. We would like to propose a similar explanation for the generic scaling of the entropy in the models of DeWolfe et al. There are two important differences. First of all, we hypothesize multi-layered pairs of pants (see figure 3 for an illustration). That is, each geometrical pair of pants is wrapped by N M2-branes rather than a single one. Secondly, the M2-branes end on Kaluza-Klein monopoles instead of on M5-branes. We claim that the entropy comes from N^3 copies of the M2-brane field theory, each with entropy $N^{3/2}$.

The symmetry of the formulae under interchange of the three four form fluxes, suggests that a picture based on string networks in Type IIB string theory might capture some of the physics. Thus, we would imagine an Argentine bola string junction, with N allowed sites for each of its ends. Each bola would consist of N strings. The M2-brane scaling of

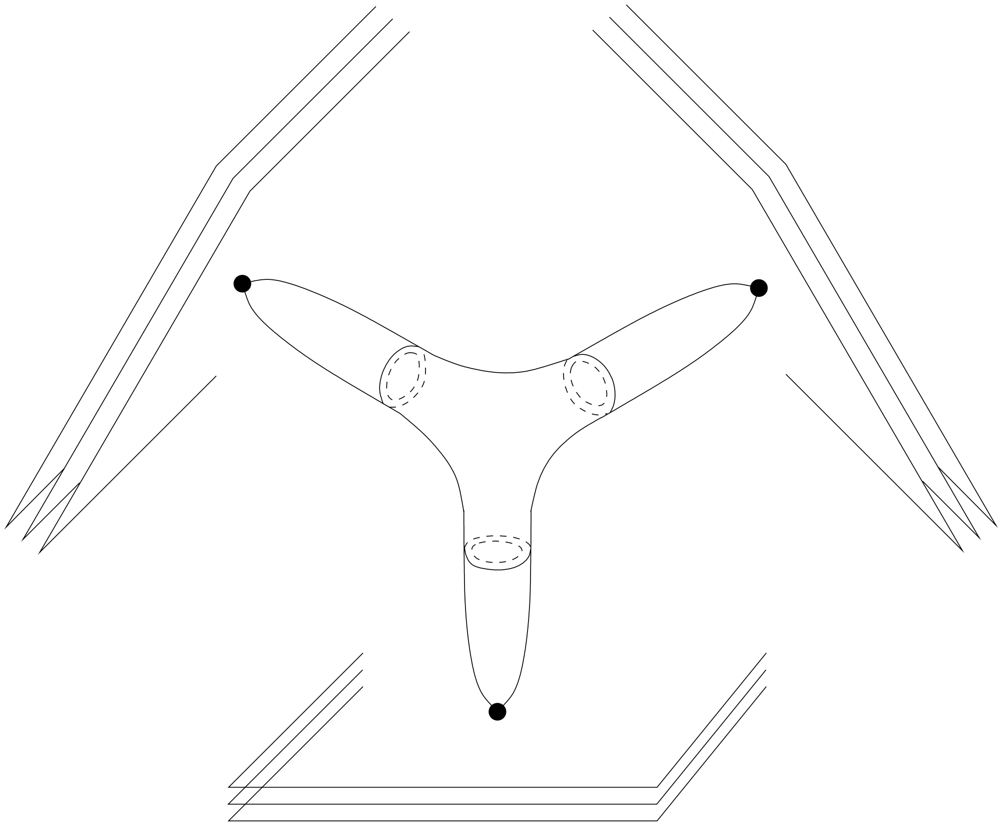

Figure 3: N M2-branes ending on 3 stacks of N Kaluza-Klein monopoles. Notice that each end of the trousers is stitched together.

the world volume theory of the string junction, could be explained by hypothesizing that it passed through a large volume on the compact manifold, where the M-theory torus had large area. The endpoints of the bola would be better described in terms of weakly coupled Type IIB string theory, though perhaps different ends would be weakly coupled in different S-duality frames.

We have argued that the weak coupling expansion claimed by DeWolfe et al. in this regime cannot be uniformly valid, since there is a region of size $N^{1/20} l_s$ where the coupling is not weak in the large N limit. Since there is no single low energy field theory that describes these configurations, and since observables in theories of gravity in Anti-de Sitter space are not local on the compact manifold, we are not sure how one would go about making a systematic computation of these observables for large N.

We presented two heuristic geometrical pictures of how the weak coupling geometry could connect on to a region best described in terms of 11D SUGRA on a manifold of weak G_2 holonomy. The approximate 4 dimensionality of the compactification is valid only in one of them. The weak coupling analysis *might* be missing a large *mushroom cap* region hidden near the strong coupling orientifold singularity. We believe that no extant methods can distinguish between these two pictures, or provide a systematic description of the physics of this system at large N. We have also presented a heuristic model of the entropy of the regime with all fluxes large. This model also depends on the existence of large regions of the compact geometry which are weakly curved eleven manifolds.

Given these arguments, and the success of our 11D picture in the regime of a single large flux where the weakly coupled region completely disappears, we see serious reasons to doubt the validity of the simple weak coupling picture advocated by DeWolfe et al., even when all fluxes are large. The reason for this is the back reaction of a large number of branes near the strongly coupled orientifold locus, which changes the geometry in ways that cannot be understood from the perturbative picture.

In our opinion, the best one could hope for would be some analog of F-theory, in which different string expansions governed local physics in different regions of the compact manifold. It is entirely unclear to us whether the particular duality frame we have emphasized is the best description of this regime. Furthermore, since we are working with a very small amount of supersymmetry, it is unlikely that we can use non-renormalization theorems to glean exact information about these compactifications from their geometrical formulation. This is a pity, because it is the only one in which an approximately 4 dimensional compactification *might* arise.

It would seem that the only way to really investigate the physics of these backgrounds of string theory is to find and solve the dual $2 + 1$ dimensional conformal field theory. For the special flux configurations described above, it is plausible that this CFT can be found by perturbing the Yang-Mills theory of D2-branes by an appropriate relevant operator, obtaining the CFT dual to M2-branes at the tip of a cone over a manifold of weak G_2 holonomy. We conjectured that the correct description of more general configurations might be explained in terms of a tensor product of field theories, or some theory which approximately reduced to such a tensor product for purposes of counting the large N asymptotics of the entropy.

Acknowledgments

One of the authors TB thanks M. Dine and A. Shomer for collaboration on an initial attempt at this project. The authors also acknowledge conversations with B. Acharya, D. Belov, F. Denef, G. Moore, S. Ramanujam, and W. Taylor, which enhanced our understanding of this difficult subject. We are particularly grateful to O. Aharony for correcting an error in the original version of the manuscript. KvdB also thanks A. Garcia-Raboso, S. Lukic and G. Torroba for useful discussions and is grateful for support and hospitality of the SCIPP where part of this work was done. Finally, we would like to thank the JHEP referee for pointing out that some of the statements in the original version of the manuscript were incorrect. This research was supported in part by DOE grant number DE-FG03-92ER40689.

A. Type IIA - Type IIB T-duality dictionary

We take the bosonic type IIA action in the string frame to be (omitting the Chern-Simons terms):

$$S_{\rm IIA} = \frac{1}{2\kappa_{10A}^2}\int\sqrt{-g_{10A}}e^{-2\varphi_A}(R+4\partial\varphi_A\partial\varphi_A) - \frac{1}{4\kappa_{10A}^2}\int e^{-2\varphi_A}H_3\wedge *H_3$$
$$-\frac{1}{4\kappa_{10A}^2}\int F_4\wedge *F_4 + F_2\wedge *F_2 + F_0\wedge *F_0\,, \tag{A.1}$$

while the bosonic type IIB action is given by:

$$S_{\rm IIB} = \frac{1}{2\kappa_{10B}^2}\int\sqrt{-g_{10B}}e^{-2\varphi_B}(R+4\partial\varphi_B\partial\varphi_B) - \frac{1}{4\kappa_{10B}^2}\int e^{-2\varphi_B}H_3\wedge *H_3$$
$$-\frac{1}{4\kappa_{10B}^2}\int \frac{1}{2}F_5\wedge *F_5 + F_3\wedge *F_3 + F_1\wedge *F_1\,, \tag{A.2}$$

where we impose the self-duality of F_5 in the equations of motion by hand (see [28] for a consistent treatment of self-dual field theories). The T-duality dictionary between both theories for a warped metric reads [29],

$$\begin{aligned}
ds_A^2 &= L_A^2 e^{2\alpha\phi}\Theta_A^2 + e^{2\beta\phi}ds_9^2 &&\leftrightarrow && ds_B^2 = L_A^{-2}e^{-2\alpha\phi}\Theta_B^2 + e^{2\beta\phi}ds_9^2\\
H_3 &= \tilde H_3 + \tilde H_2\wedge\Theta_A &&\leftrightarrow && H_3 = \tilde H_3 - \tilde F^{NS}\wedge\Theta_B\\
&\varphi_A &&\leftrightarrow && \varphi_B = \varphi_A - \alpha\phi\,,\\
F_4 &= \tilde F_4 + \tilde F_3\wedge L_A\Theta_A &&\leftrightarrow && F_5 = e^{(\alpha+\beta)\phi} *_9 \tilde F_4 + \tilde F_4\wedge L_A^{-1}\Theta_B\\
F_2 &= \tilde F_2 + \tilde F_1\wedge L_A\Theta_A &&\leftrightarrow && F_3 = \tilde F_3 + \tilde F_2\wedge L_A^{-1}\Theta_B\\
F_0 &= \tilde F_0 &&\leftrightarrow && F_1 = \tilde F_1 + \tilde F_0\wedge L_A^{-1}\Theta_B\\
&\frac{1}{2\kappa_{10A}^2} &&\leftrightarrow && \frac{1}{2\kappa_{10B}^2} = \frac{L_A^2}{2\kappa_{10A}^2}\,,\\
d\Theta_A &= \tilde F^{NS} &&\leftrightarrow && d\Theta_B = -\tilde H_2\,,\\
\Theta_A &= 2\pi\sqrt{\alpha'}dx + \tilde A^{NS} &&\leftrightarrow && \Theta_B = 2\pi\sqrt{\alpha'}dx - \tilde B_1\,,
\end{aligned} \tag{A.3}$$

where the last line is valid locally, with $\tilde F^{NS} = d\tilde A^{NS}$, $\tilde H_2 = d\tilde B_1$ and $x\in[0,1]$ parametrizes the U(1) isometry. The Hodge star in F_5 is with respect to the ds_9^2 metric on the type IIB side of the dictionary. In our notation, the forms on the right hand sides of the equations never contain Θ_A or Θ_B explicitly.

B. Computing the double T-dual of the DeWolfe et al. background

We start from the DeWolfe et al. solution (2.1)–(2.8). We use the dictionary from appendix A. To apply a first T-duality in the x_1-direction, $x_1\in[0,1]$, we take,

$$L_A^2 = \frac{\gamma_1}{4\pi^2\alpha'}9^{-\frac{1}{3}} \tag{B.1}$$
$$\Theta_A = 2\pi\sqrt{\alpha'}\,9^{\frac{1}{6}}\,dx_1 \tag{B.2}$$
$$\Theta_B = 9^{\frac{1}{6}}\,\Theta_1 \tag{B.3}$$
$$\alpha=\beta=0\,. \tag{B.4}$$

The factor $9^{1/6}$ comes from the discrete symmetries $\mathbb{Z}_3 \times \mathbb{Z}_3$ (see (2.13)). This results in:

$$ds^2 = \frac{4\pi^2\alpha'}{\gamma_1} 9^{\frac{2}{3}}\,\Theta_1^2 + \gamma_1 dx_2^2 + \gamma_2(dx_3^2 + dx_4^2) + \gamma_3(dx_5^2 + dx_6^2) + ds^2_{\mathrm{AdS}_4} \tag{B.5}$$

$$H_3 = 4\pi^2\alpha' h_3 \sqrt[4]{3}\sqrt{2}\,(dx_2 \wedge dx_3 \wedge dx_6 + dx_2 \wedge dx_4 \wedge dx_5) \tag{B.6}$$

$$e^{\varphi_B} = \frac{1}{4}|h_3|\sqrt[4]{\frac{3^3 5}{|f_0 f_4^1 f_4^2 f_4^3|}} \tag{B.7}$$

$$\begin{aligned} F_5 &= 4(2\pi\sqrt{\alpha'})^3 \sqrt[3]{3} f_4^1 \Big(*_9(dx_3 \wedge dx_4 \wedge dx_5 \wedge dx_6) \\ &\qquad + \sqrt{\frac{4\pi^2\alpha'\, 9^{\frac{2}{3}}}{\gamma_1}} dx_3 \wedge dx_4 \wedge dx_5 \wedge dx_6 \wedge \Theta_1 \Big) \end{aligned} \tag{B.8}$$

$$\begin{aligned} &= \left(4(2\pi\sqrt{\alpha'})^3 \sqrt[3]{3} f_4^1 \frac{1}{\gamma_2\gamma_3} \mathrm{vol}_4\right) \sqrt{\frac{\gamma_1}{4\pi^2\alpha'\, 4 \cdot 3^{-\frac{1}{3}}}} \wedge (2\pi\sqrt{\alpha'}\, 2 \cdot 3^{-\frac{1}{6}} dx_2) \\ &\quad + *_{\tilde{9}} \left(4(2\pi\sqrt{\alpha'})^3 \sqrt[3]{3} f_4^1 \frac{1}{\gamma_2\gamma_3} \mathrm{vol}_4\right) \end{aligned} \tag{B.9}$$

$$F_3 = -4(2\pi\sqrt{\alpha'})^2 \sqrt[3]{3} \sqrt{\frac{4\pi^2\alpha'}{\gamma_1}} (f_4^2\, dx_5 \wedge dx_6 \wedge dx_2 + f_4^3\, dx_2 \wedge dx_3 \wedge dx_4) \tag{B.10}$$

$$F_1 = \frac{f_0}{\sqrt{\gamma_1}} 9^{\frac{1}{3}}\,\Theta_1 \tag{B.11}$$

$$\frac{1}{2\kappa_{10B}^2} = \frac{1}{2\kappa_{10A}^2} \frac{\gamma_1}{4\pi^2\alpha'\, 9^{\frac{1}{3}}} \tag{B.12}$$

$$\Theta_1 = 2\pi\sqrt{\alpha'} dx_1 + 2\pi\sqrt{\alpha'} h_3 \frac{\sqrt[4]{3}\sqrt{2}}{9^{\frac{1}{3}}} (x_3 dx_5 - x_4 dx_6)\,. \tag{B.13}$$

The last line is again only valid locally. We use the notation where, $*_9$, means the Hodge star with respect to the metric after T-duality without the x_1-direction, while, $*_{\tilde{9}}$, means the Hodge star with respect to the metric after T-duality without the x_2-direction. vol_4 is the volume form of AdS_4.

The first T-duality transformation splits the original orientifold (2.17) into an O5- and an O7-plane:

$$\mathrm{O5}: \frac{\sqrt[4]{3}\sqrt{2}}{2\pi\sqrt{\alpha'}\, 9^{\frac{1}{6}}} (+dx_3 \wedge dx_5 - dx_4 \wedge dx_6) \tag{B.14}$$

$$\mathrm{O7}: \frac{\sqrt[4]{3}\sqrt{2}\, 9^{\frac{1}{6}}}{2\pi\sqrt{\alpha'}\, 2 \cdot 3^{-\frac{1}{6}}} (-dx_4 \wedge dx_5 - dx_3 \wedge dx_6) \wedge (2\pi\sqrt{\alpha'}\, 2 \cdot 3^{-\frac{1}{6}} dx_2) \wedge \Theta_1 \tag{B.15}$$

After this first T-duality the solution still has an (approximate) U(1)-isometry in the x_2-direction, $x_2 \in [0, \sqrt{3}/2]$. We take

$$L_A^2 = \frac{4\pi^2\alpha'}{\gamma_1} \left(\frac{4}{3} 9^{\frac{1}{3}}\right) \tag{B.16}$$

$$\Theta_B = 2\pi\sqrt{\alpha'} \left(\frac{2}{\sqrt{3}} 9^{\frac{1}{6}}\right) dx_2 \tag{B.17}$$

$$\Theta_A = 2 \cdot 3^{-\frac{1}{6}}\,\Theta_2 \tag{B.18}$$

$$\alpha = \beta = 0\,, \tag{B.19}$$

this results in the solution (3.28)–(3.36).

C. Type IIA - M-theory dictionary

For the type IIA theory we start again from the action (A.1), for M-theory we take as definition of our theory,

$$S_{\mathrm{M}} = \frac{1}{2\kappa_{11}^2}\int \sqrt{-g_{11}}R - \frac{1}{4\kappa_{11}^2}\int F_4 \wedge *F_4 \tag{C.1}$$
$$-\frac{1}{4\kappa_{11}^2}\int C_3 \wedge F_4 \wedge F_4 \,. \tag{C.2}$$

The compactification of M-theory on a circle gives the following type IIA - M-theory correspondence:

$$\begin{array}{lcrl} ds_A^2 = ds_{10}^2 & \leftrightarrow & ds_M^2 &= L_M^2 e^{\frac{4}{3}\varphi_A}\Theta_M^2 + e^{-\frac{2}{3}\varphi_A} ds_{10}^2 \\ \varphi_A & & & \\ H_3 & \leftrightarrow & G_4 &= F_4 + H_3 \wedge L_M \Theta_M \\ F_4 & & & \\ F_2 & \leftrightarrow & d\Theta_M &= \frac{1}{L_M}F_2 \,, \\ & & \Theta_M &= dx_M + \frac{1}{L_M}C_1 \,, \\ F_0 = 0 & & & \\ \frac{1}{2\kappa_{10A}^2} & \leftrightarrow & \frac{1}{2\kappa_{11M}^2} &= \frac{1}{2\kappa_{10A}^2 L_M} \,, \\ & & L_M &= 2\pi \frac{\kappa_{10A}}{\sqrt{\pi}(2\pi\sqrt{\alpha'})^3} \,, \end{array} \tag{C.3}$$

with $x_M \in [0, 1]$.

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